

PROPAGATION OF LOVE SURFACE WAVES IN AN ELASTIC DIATOMIC MEDIUM

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Abstract—Love waves propagating over the surface of a half-space covered by a layer of different diatomic materials are examined. After establishing the associated constitutive equations the equations of motion are derived, and the coefficients of the wave attenuation normal to the direction of propagation found. Fulfillment of the boundary conditions at a free surface and the interface leads to a dispersion equation in the form of a vanishing determinant of the sixth order. To obtain more tangible results, an example involving relaxed bonds between the constituents of the diatomic materials is given, and illustrated by a graph displaying the dependence of the phase velocity on the wave number in the first wave mode.

1. INTRODUCTION

The idea that a bare homogeneous continuum may serve as a model of a real material was, and still is, so successful in practical applications that, only after the advent of such highly nonhomogeneous materials as composites, the developers of mechanics started to give serious consideration to the internal, discreet, structure of matter.

The modern efforts to incorporate some kind of microstructure into the traditional classical model were to a great extent inspired by the renowned treatise of Truesdell and Toupin (1960), in which among others the forgotten concept of the couple stress was comprehensively analysed.

First, the theory of polar media, most of all in the writings of Mindlin (e.g., 1964) and Eringen (e.g., 1968), was developed. In this model, known as a micropolar and a micromorphic one, to the material particles is ascribed some freedom of motions independent of the motions of the body as a whole.

Independently, Green and Rivlin (1964) proposed a theory of great generality. It includes concepts of higher order gradients of deformation and velocity as well as of multipolar forces.

A third line of refining the classical model involved the idea of the nonlocal particle interactions. This concept novel in Mechanics of Materials was, of course, a household word in atomic physics, in particular in the Lattice Dynamics of Born and Kármán in which the classical elasticity was derived from purely atomistic assumptions. The nonlocal aspect of continuum was discussed by Kroener (1969), Eringen (1974) and Kunin (1988) among others.

In a somewhat different direction progressed the paper by Demiray (1973) on mechanics of polyatomic bodies. In this model a particle is composed of several subparticles (atoms, say). The subparticles interact with each other and have different thermomechanical characteristics. They overlap in initial positions but are carried by the deformation into different spatial positions. In detail Demiray's theory was derived for a diatomic body.

In recent years combinations of particular models were suggested; for example, amalgams of nonlocality with micropolarity (Eringen, 1976), polyatomicity (Demiray, 1977) and viscoelasticity (Ahmadi, 1975; Nowinski, 1986), respectively, as well as with polyatomicity and viscoelasticity (Demiray, 1982).

In the present paper Love waves propagating over a layer covering a half-space of different diatomic materials are examined. After establishing the associated constitutive equations, the equations of motion are derived, and the coefficients of the wave attenuation normal to the direction of propagation found. Fulfillment of the boundary conditions at a free surface and the interface leads to a dispersion equation in the form of a vanishing

determinant of the sixth order. To obtain more tangible results an example involving relaxed bonds between the constituents of the diatomic materials is given and illustrated by a graph depicting the dependence of the phase velocity on the wave number in the first wave mode.

2. FUNDAMENTAL EQUATIONS

Consider a half-infinite space referred to a Cartesian coordinate system x_1, x_2, x_3 such that the x_1x_3 plane coincides with the plane boundary of the half-space, and the x_2 -axis points into the interior of the half-space. The latter is covered by an infinite layer of thickness h , the material of which differs from the material of the half-space. Both materials, of the layer (marked L), and of the half-space (marked S), are assumed to be elastic, isotropic, homogeneous and of the diatomic structure. The two constituents of each material are designated by the index in parentheses, (1) or (2).

Imagine now a plane surface wave propagating over the structure in question in the x_1 -direction so that all kinematic and dynamic quantities involved are automatically independent of the coordinate x_3 . The wave is assumed to be a horizontally polarized shear wave, so that $u_1^{(\alpha)} = u_2^{(\alpha)} \equiv 0$ and

$$u_3^{(\alpha)} = F^{(\alpha)}(x_2) e^{ik(x_1 - \omega t)}. \quad (1)$$

Here $\alpha = 1, 2$, $F^{(\alpha)}$ is a function of x_2 to be determined later, k is the wave number, t is the time and ω is the circular frequency. If this is so, then the only not identically vanishing strain components are $e_{23}^{(\alpha)} = e_{32}^{(\alpha)} = \frac{1}{2}u_{3,2}^{(\alpha)}$ and $e_{31}^{(\alpha)} = e_{13}^{(\alpha)} = \frac{1}{2}u_{3,1}^{(\alpha)}$, where the comma denotes a partial coordinate differentiation (e.g. $u_{3,2} = \partial u_3(x_1, x_2)/\partial x_2$).

In the context of the diatomic theory the only not identically vanishing components of the (asymmetric) stress tensors are

$$\begin{aligned} \tau_{31}^{(1)} &= a_2 u_{3,1}^{(1)} + a_6 u_{3,1}^{(2)} + a_7 (u_{3,1}^{(1)} - u_{3,1}^{(2)}), \\ \tau_{13}^{(1)} &= a_2 u_{3,1}^{(1)} + a_6 u_{3,1}^{(2)} + a_7 (u_{3,1}^{(2)} - u_{3,1}^{(1)}), \\ \tau_{32}^{(1)} &= a_2 u_{3,2}^{(1)} + a_6 u_{3,2}^{(2)} + a_7 (u_{3,2}^{(1)} - u_{3,2}^{(2)}), \\ \tau_{23}^{(1)} &= a_2 u_{3,2}^{(1)} + a_6 u_{3,2}^{(2)} + a_7 (u_{3,2}^{(2)} - u_{3,2}^{(1)}), \\ \tau_{31}^{(2)} &= a_4 u_{3,1}^{(2)} + a_6 u_{3,1}^{(1)} + a_7 (u_{3,1}^{(2)} - u_{3,1}^{(1)}), \\ \tau_{13}^{(2)} &= a_4 u_{3,1}^{(2)} + a_6 u_{3,1}^{(1)} + a_7 (u_{3,1}^{(1)} - u_{3,1}^{(2)}), \\ \tau_{32}^{(2)} &= a_4 u_{3,2}^{(2)} + a_6 u_{3,2}^{(1)} + a_7 (u_{3,2}^{(2)} - u_{3,2}^{(1)}), \\ \tau_{23}^{(2)} &= a_4 u_{3,2}^{(2)} + a_6 u_{3,2}^{(1)} + a_7 (u_{3,2}^{(1)} - u_{3,2}^{(2)}), \end{aligned} \quad (2)$$

where a denotes material coefficients.

The equations of motion are now

$$\begin{aligned} \tau_{ki,k}^{(1)} - \rho_1 a_i^{(1)} + R_i &= 0, \\ \tau_{ki,k}^{(2)} - \rho_2 a_i^{(2)} + R_i &= 0, \end{aligned} \quad (3)$$

with a_i as the acceleration, $R_i = (u_i^{(2)} - u_i^{(1)})a_0$ as the rate of momentum transfer between the constituents (or the mutual force), a_0 as a coefficient and ρ as the mass density.†

In the case under consideration we have

† The significance of R_i (or a_0 , respectively) is discussed by Demiray (1973).

$$\begin{aligned} \tau_{13,1}^{(1)} + \tau_{23,2}^{(1)} - \rho_1 \ddot{u}_3^{(1)} + a_0(u_3^{(2)} - u_3^{(1)}) &= 0, \\ \tau_{13,1}^{(2)} + \tau_{23,2}^{(2)} - \rho_2 \ddot{u}_3^{(2)} + a_0(u_3^{(1)} - u_3^{(2)}) &= 0, \end{aligned} \tag{4}$$

so that substitution of eqns (2) into (4) gives after some manipulations,

$$\begin{aligned} b_4 \nabla u_3^{(1)} + b_5 \nabla u_3^{(2)} + b_0(u_3^{(2)} - u_3^{(1)}) - \rho_1 \ddot{u}_3^{(1)} &= 0, \\ b_6 \nabla u_3^{(2)} + b_5 \nabla u_3^{(1)} + b_0(u_3^{(1)} - u_3^{(2)}) - \rho_2 \ddot{u}_3^{(2)} &= 0, \end{aligned} \tag{5}$$

where

$$\nabla = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2}, \quad b_4 = a_2 - a_7, \quad b_5 = a_6 + a_7, \quad b_6 = a_4 - a_7, \quad b_0 = a_0,$$

and the overdot denotes time differentiation. It is of interest to observe the correspondence existing between the material diatomic coefficients and their classical Lamé counterparts (Table 1). It is apparent from Table 1 [cf. also Nowinski (1989)] that the diatomic coefficients of Group I play the roles of the Lamé shear moduli and reduce to the latter if the interactions between the constituent media are severed. The interaction coefficients of Group II on the other hand are absent in the classical approach, and by suppressing them one arrives at the classical counterparts of eqns (2) through (5).

Taking advantage of the eqn (1) it is a straightforward matter to decouple the field equations (5), and arrive at the following differential equations:

$$\begin{aligned} F_{,2222}^{(1)} + \lambda_1 F_{,22}^{(1)} - \lambda_2 F^{(1)} &= 0, \\ F_{,2222}^{(2)} + \lambda_3 F_{,22}^{(2)} - \lambda_4 F^{(2)} &= 0, \end{aligned} \tag{6}$$

with the notation

$$\begin{aligned} \lambda_1 &= \frac{b_4}{b_5 \mu_1} + \frac{\mu_2}{\mu_1} - \frac{d}{b_5}, & \lambda_2 &= \frac{c_1}{b_5 \mu_1} + \frac{d \mu_2}{b_5 \mu_1}, \\ \lambda_3 &= \frac{b_6}{b_5 \mu_3} + \frac{\mu_4}{\mu_3} - \frac{d}{b_5}, & \lambda_4 &= \frac{c_2}{b_5 \mu_3} + \frac{d \mu_4}{\mu_3}, \\ \mu_1 &= \frac{b_4 b_6 - b_5^2}{b_6 d - b_5 c_2}, & \mu_2 &= \frac{b_5 d - b_6 c_1}{b_6 d - b_5 c_2}, & \mu_3 &= \frac{b_4 b_6 - b_5^2}{b_4 d - b_5 c_1}, \\ \mu_4 &= \frac{b_5 d - b_4 c_2}{b_4 d - b_5 c_1}, & c_1 &= b_4 k^2 + b_0 - \rho_1 \omega^2, & c_2 &= b_6 k^2 + b_0 - \rho_2 \omega^2, \\ d &= b_5 k^2 - b_0; & b_4 &= a_2 - a_7, & b_5 &= a_6 + a_7, & b_6 &= a_4 - a_7, & b_0 &= a_0. \end{aligned} \tag{7}$$

Solutions to eqns (6) appropriate for the purposes at hand are: for the first constituent of the layer,

Table 1. Diatomic and monoatomic coefficients

Group	Diatomic coefficients	Monoatomic moduli
I. Constitutional coefficients	a_2	μ_1
	a_4	μ_2
II. Interaction coefficients	a_6	—
	a_7	—
	a_0	—

$$F_1(x_2) = A_1 e^{-iax_2} + B_1 e^{iax_2}; \tag{8a}$$

for the second constituent of the layer,

$$F_2(x_2) = A_2 e^{-ibx_2} + B_2 e^{ibx_2}, \tag{8b}$$

where

$$\begin{aligned} a^2 &= [\frac{1}{2}(\lambda_1 \pm \sqrt{\lambda_1^2 + 4\lambda_2})]^{1/2}, \\ b^2 &= [\frac{1}{2}(\lambda_3 \pm \sqrt{\lambda_3^2 + 4\lambda_4})]^{1/2}. \end{aligned} \tag{9}$$

A_i and B_i ($i = 1, 2$) are constants to be determined later, and for the sake of argument we assume that the coefficients a and b are real and positive.

3. BOUNDARY CONDITIONS

At this stage it is necessary to consider the boundary conditions. Before doing this it is convenient to write down the equations for the displacements, both in the layer and in the half-space, in the form consistent with the physical aspects of the problem. The latter—as it is well known—require concentration of the wave energy in the layer, and a rapid decay of the motions in the half-space.

In this connection we assume that :

In the layer, ($-h \leq x_2 \leq 0$)

First constituent

$$u_3^{(1)L} = [A_1^L e^{-ia_L x_2} + B_1^L e^{ia_L x_2}] e^{-i(kx_1 - \omega t)}.$$

Second constituent

$$u_3^{(2)L} = [A_2^L e^{-ib_L x_2} + B_2^L e^{ib_L x_2}] e^{-i(kx_1 - \omega t)}. \tag{10}$$

In the half-space, ($0 \leq x_2$)

First constituent

$$u_3^{(1)S} = [C_1^S e^{-ia_S x_2}] e^{-i(kx_1 - \omega t)}.$$

Second constituent

$$u_3^{(2)S} = [C_2^S e^{-ib_S x_2}] e^{-i(kx_1 - \omega t)}. \tag{11}$$

We note that in the problem in hand the surface $x_2 = -h$ is free from load, and at the common boundary, $x_2 = 0$, of the layer and the half-space, the continuity of the stresses and the displacements is to be secured. Since the only stress component acting in the planes perpendicular to the x_2 -axis is the component τ_{23} , we have

$$\tau_{23}^L = 0 \quad \text{at} \quad x_2 = -h, \tag{12a}$$

$$\tau_{23}^L = \tau_{23}^S \tag{12b}$$

$$u_3^L = u_3^S \tag{12c}$$

for all values of x_1 and t .

It should be clear that in the problem examined the existence of six unknown coefficients, that is, of $A_1^L, A_2^L, B_1^L, B_2^L, C_1^S$ and C_2^S , stipulates the existence of six boundary conditions. This can be done if we demand the three conditions (12) to be obeyed by each constituent of the materials of the layer and the half-space. The catch here, however, is that

there is no unique way to satisfy the last named requirement with regard to the conditions (12b) and (12c). For example, one may demand that

$$\begin{aligned} & \tau_{23}^{(1)L} = \tau_{23}^{(1)S}, \quad u_3^{(1)L} = u_3^{(1)S}, \\ \text{and} \quad & \tau_{23}^{(2)L} = \tau_{23}^{(2)S}, \quad u_3^{(2)L} = u_3^{(2)S}, \end{aligned} \quad \text{at } x_2 = 0. \quad (13a)$$

One may also demand that

$$\begin{aligned} & \tau_{23}^{(1)L} = \tau_{23}^{(2)S}, \quad u_3^{(1)L} = u_3^{(2)S}, \\ \text{and} \quad & \tau_{23}^{(2)L} = \tau_{23}^{(1)S}, \quad u_3^{(2)L} = u_3^{(1)S}, \end{aligned} \quad \text{at } x_2 = 0, \quad (13b)$$

or one may select another combination, different from those given above.† In what follows we adhere to the scheme (13a), and as a result the set of boundary conditions becomes

for the first component of the layer,‡

$$(a_2^L - a_7^L)(-a_L e^{ia_L h} A_1^L + a_L e^{-ia_L h} B_1^L) + (a_6^L + a_7^L)(-b_L e^{ib_L h} A_2^L + b_L e^{-ib_L h} B_2^L) = 0, \quad (14a)$$

for the first components of the layer and the half-space

$$A_1^L + B_1^L = C_1^S, \quad (14b)$$

$$\begin{aligned} (a_2^L - a_7^L)(-a_L A_1^L + a_L B_1^L) + (a_6^L + a_7^L)(-b_L A_2^L + b_L B_2^L) \\ + (a_2^S - a_7^S)a_S C_1^S + (a_6^S - a_7^S)b_S C_2^S = 0; \end{aligned} \quad (14c)$$

for the second component of the layer,

$$(a_4^L - a_7^L)(-b_L e^{ib_L h} A_2^L + b_L B_2^L e^{-ib_L h}) + (a_6^L + a_7^L)(-a_L e^{ia_L h} A_1^L + a_L e^{-ia_L h} B_1^L) = 0, \quad (15a)$$

for the second components of the layer and the half-space,

$$A_2^L + B_2^L = C_2^S, \quad (15b)$$

$$\begin{aligned} (a_4^L - a_7^L)(-b_L A_2^L + b_L B_2^L) + (a_6^L + a_7^L)(-a_L A_1^L + a_L B_1^L) \\ + (a_4^S - a_7^S)b_S C_2^S + (a_6^S + a_7^S)a_S C_1^S = 0. \end{aligned} \quad (15c)$$

It goes almost without saying that the existence of a nontrivial solution of the foregoing system of six linear and homogeneous algebraic equations in six unknown coefficients implies vanishing of the principal determinant of the system. This means that

† This is not the only peculiar feature of the diatomic theory. There is, for instance, also no unique way of selecting the strain measures, so that all measures consistent with the principle of objectivity are admissible [cf. Demiray (1982)].

‡ It is recalled that a_L and b_L (a_S and b_S) denote the coefficients (9) for the 1st and 2nd components of the layer (of the half-space), respectively.

$$\begin{vmatrix}
 m_{11} & m_{12} & m_{13} & m_{14} & 0 & 0 \\
 1 & 1 & 0 & 0 & -1 & 0 \\
 m_{31} & m_{32} & m_{33} & m_{34} & m_{35} & m_{36} \\
 m_{41} & m_{42} & m_{43} & m_{44} & 0 & 0 \\
 0 & 0 & 1 & 1 & 0 & -1 \\
 m_{61} & m_{62} & m_{63} & m_{64} & m_{65} & m_{66}
 \end{vmatrix} = 0, \quad (16)$$

where

$$\begin{aligned}
 m_{11} &= -(a_2^L - a_7^L)a_L e^{ia_L h}, & m_{41} &= -(a_6^L + a_7^L)a_L e^{ia_L h}, \\
 m_{12} &= (a_2^L - a_7^L)a_L e^{-ia_L h}, & m_{42} &= (a_6^L + a_7^L)a_L e^{-ia_L h}, \\
 m_{13} &= -(a_6^L + a_7^L)b_L e^{ib_L h}, & m_{43} &= -(a_4^L - a_7^L)b_L e^{ib_L h}, \\
 m_{14} &= (a_6^L + a_7^L)b_L e^{-ib_L h}, & m_{44} &= -(a_4^L + a_7^L)b_L e^{-ib_L h}, \\
 m_{31} &= -(a_2^L - a_7^L)a_L, & m_{61} &= -(a_6^L + a_7^L)a_L, \\
 m_{32} &= (a_2^L - a_7^L)a_L, & m_{62} &= (a_6^L + a_7^L)a_L, \\
 m_{33} &= -(a_6^L - a_7^L)b_L, & m_{63} &= -(a_4^L + a_7^L)b_L, \\
 m_{34} &= (a_6^L + a_7^L)b_L, & m_{64} &= (a_4^L + a_7^L)b_L, \\
 m_{35} &= (a_2^S - a_7^S)a_S, & m_{65} &= (a_6^S + a_7^S)a_S, \\
 m_{36} &= (a_6^S + a_7^S)b_S, & m_{66} &= (a_4^S - a_7^S)b_S.
 \end{aligned}$$

It is not difficult to convince oneself that, generally speaking, eqn (16) implicitly relating the wave number, k , to the wave frequency, ω , manifests the dispersive character of the wave motion. Thus, Love waves in diatomic media, exactly like their monoatomic counterparts, turn out to be subject to dispersion. It need hardly be added, however, that due to the intricacy of the dispersion equation (16) as well as to the presence of a great number of little known material coefficients a general analysis of the equation would not only present serious computational difficulties, but along with this would obscure rather than clarify the main points of the problem.

In this connection, in the illustrative example examined in the subsequent section we wish to consider a somewhat simplified model of the diatomic material. It is defined as follows:

(a) There exist relatively weak bonds between the constituents, such that the coefficients

$$a_6 = a_7 = 0, \quad (17a)$$

and the remaining bond coefficient b_0 is such that

$$(b_0)^2 \ll b_0. \quad (17b)$$

This means that the only intrinsic bond retained is the one that is associated with the transfer of momentum between the constituents, and its value is limited to the term of the first order.

(b) The particular bond coefficients as well as the mass densities are respectively equal. This means that

$$b_{01}^L = b_{02}^L = b_{01}^S = b_{02}^S = b_0, \tag{18a}$$

$$\rho_1^L = \rho_2^L = \rho_1^S = \rho_2^S = \rho. \tag{18b}$$

(c) The associated shear moduli a_2^L, a_4^L, a_2^S and a_4^S , are all different.

4. ILLUSTRATIVE EXAMPLE

With the restrictions (a) through (c) listed in Section 2 in mind we set $a_6 = a_7 = 0$, and from the relations (16) find that coincidentally $m_{13} = m_{14} = m_{33} = m_{34} = m_{36} = m_{41} = m_{42} = m_{61} = m_{62} = m_{65} = 0$. This being so, eqn (16) splits into two separate dispersion equations

$$\tan a_L h = \frac{ia_2^S a_S}{a_2^L a_L}, \tag{19}$$

and

$$\tan b_L h = \frac{ia_4^S b_S}{a_4^L b_L}, \tag{20}$$

which connect (as already mentioned) the phase velocity $c = \omega/k$ with the wave number, k (or wavelength, $= 2\pi/k$). It is not difficult to verify that if the diatomic medium becomes converted into the classical monoatomic one, then both conditions (19) and (20) reduce to the well-known and extensively discussed Love's equation [cf. H. Nowinski (1986) eqn (4.212)†].

In order to determine the quantities a_L, b_L and a_S, b_S it is more convenient to go back to eqns (5). We easily find that as regards the layer there is for both its first and second constituents

$$(a_L)^2 = (b_L)^2 = \frac{1}{2} \{ -\sigma_1^L \pm [(\sigma_1^L)^2 - 4\sigma_2^L]^{1/2} \}, \tag{21a}$$

where

$$\sigma_1^L = \frac{a_4^L c_1^L + a_2^L c_2^L}{a_2^L a_4^L}, \quad \sigma_2^L = \frac{c_1^L c_2^L - (b_0)^2}{a_2^L a_4^L}. \tag{21b}$$

Again, as regards the half-space

$$(a_S)^2 = (b_S)^2 = \frac{1}{2} \{ -\sigma_1^S \pm [(\sigma_1^S)^2 - 4\sigma_2^S]^{1/2} \}, \tag{22}$$

where

$$\sigma_1^S = \frac{a_4^S c_1^S + a_2^S c_2^S}{a_2^S a_4^S}, \quad \sigma_2^S = \frac{c_1^S c_2^S - (b_0)^2}{a_2^S a_4^S}. \tag{23a}$$

Also

$$\begin{aligned} c_1^L &= k^2 a_2^L \left[1 - \frac{\rho \omega^2 - b_0}{k^2 a_2^L} \right], & c_2^L &= k^2 a_4^L \left[1 - \frac{\rho \omega^2 - b_0}{k^2 a_4^L} \right], \\ c_1^S &= k^2 a_2^S \left[1 - \frac{\rho \omega^2 - b_0}{k^2 a_2^S} \right], & c_2^S &= k^2 a_4^S \left[1 - \frac{\rho \omega^2 - b_0}{k^2 a_4^S} \right]. \end{aligned} \tag{23b}$$

A simple calculation with the assumptions (18) and (19) in mind yields two values of the exponents a_L and a_S , respectively, in particular,

† See also the remarks following eqn (27).

$$a_L = b_L = \begin{cases} \left(-\frac{c_1^L}{a_2^L}\right)^{1/2} = k \left[\frac{\rho\omega^2 - b_0}{k^2 a_2^L} - 1 \right]^{1/2}, \\ \left(-\frac{c_2^L}{a_4^L}\right)^{1/2} = k \left[\frac{\rho\omega^2 - b_0}{k^2 a_4^L} - 1 \right]^{1/2}, \end{cases} \quad (24a)$$

and

$$a_S = b_S = \begin{cases} \left(-\frac{c_1^S}{a_2^S}\right)^{1/2} = k \left[\frac{\rho\omega^2 - b_0}{k^2 a_2^S} - 1 \right]^{1/2}, \\ \left(-\frac{c_2^S}{a_4^S}\right)^{1/2} = k \left[\frac{\rho\omega^2 - b_0}{k^2 a_4^S} - 1 \right]^{1/2}. \end{cases} \quad (24b)$$

We introduce the notation defining the wave quasi-frequency, Ω , by

$$\rho\omega^2 - b_0 \equiv \rho\Omega^2, \quad (25)$$

so that $\omega^2 = \Omega^2 + b_0/\rho$. We rewrite eqns (24) in the form

$$a_L = b_L = \begin{cases} k \left[\left(\frac{c^*}{c_{L1}}\right)^2 - 1 \right]^{1/2}, \\ k \left[\left(\frac{c^*}{c_{L2}}\right)^2 - 1 \right]^{1/2}, \end{cases} \quad (26a)$$

$$a_S = b_S = \begin{cases} k \left[\left(\frac{c^*}{c_{S1}}\right)^2 - 1 \right]^{1/2}, \\ k \left[\left(\frac{c^*}{c_{S2}}\right)^2 - 1 \right]^{1/2}, \end{cases} \quad (26b)$$

where

$$(c^*)^2 = \Omega^2/k^2, \quad c_{L1} = (a_2^L/\rho)^{1/2}, \quad c_{L2} = (a_4^L/\rho)^{1/2}, \quad c_{S1} = (a_2^S/\rho)^{1/2}, \quad c_{S2} = (a_4^S/\rho)^{1/2}. \quad (26c)$$

We observe that the last four relations define the wave velocities in the material of the first and second constituents of the layer and the half-space, respectively. This completes the solution of the problem, and with all the preliminaries out of the way we are now in a position to write down the dispersion equations (19) and (20) in the following final form:

$$\tan kh \left[\left(\frac{c^*}{c_{L1}}\right)^2 - 1 \right]^{1/2} = a_2^S \left[1 - \left(\frac{c^*}{c_{S1}}\right)^2 \right]^{1/2} / a_2^L \left[\left(\frac{c^*}{c_{L1}}\right)^2 - 1 \right]^{1/2}, \quad (27a)$$

$$\tan kh \left[\left(\frac{c^*}{c_{L2}}\right)^2 - 1 \right]^{1/2} = a_4^S \left[1 - \left(\frac{c^*}{c_{S2}}\right)^2 \right]^{1/2} / a_4^L \left[\left(\frac{c^*}{c_{L2}}\right)^2 - 1 \right]^{1/2}. \quad (27b)$$

It is immediately seen that in this form the dispersion equations (27a) and (27b) do not structurally differ from those of the classical elasticity, and some of the conclusions

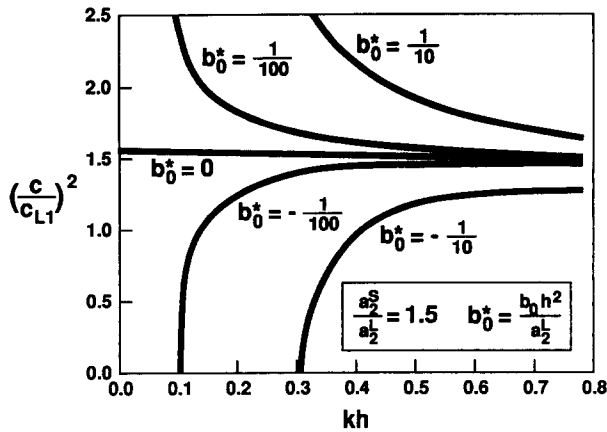


Fig. 1. Square of the relative velocity versus the nondimensional wave number.

drawn in the monoatomic case may, therefore, be repeated *mutatis mutandis* in the diatomic case.

(1) First, that the real values of the velocity c^* lie in the intervals $c_{L1} < c^* \leq c_{S1}$ and $c_{L2} < c^* \leq c_{S2}$, where the actual bounds depend on the relative values of the material moduli a_2^s , a_4^l , a_2^s and a_4^s .

(2) Second, because of the periodicity of the tangent function there exists an infinite number of wave modes.

In other respects the diatomic case differs radically from its monoatomic counterpart. First, as already noted in Section 2, there is no unique solution to the given problem in as much as the boundary conditions may be formulated in different ways. Second, for the material in question the dispersion curve in each particular mode may possess up to four distinct branches. This is in contrast to the classical elasticity in which a unique dispersion branch exists.

Figure 1 illustrates one case of the dependence of the square of the actual relative velocity, c/c_{L1} , on the nondimensional wave number kh for several values of the bond parameter $b_0^* = b_0 h^2 / a_2^l$, all for the fixed ratio $a_2^s / a_2^l = 1.5$. It is seen that the existence of the interatomic bond symbolized by the coefficient b_0 radically changes the course of the curves, and makes the wave velocity become unbounded as the wave number tends to zero.

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